# The flavor physics in unified gauge theory from an $S_3 \times \mathcal{P}$ discrete symmetry

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### Abstract

We investigate the phenomenological implication of the discrete symmetry $S_3 \times \mathcal{P}$ on flavor physics in $SO(10)$ unified theory. We construct a minimal renormalizable model which reproduce all the masses and mixing angle of both quarks and leptons. As usually the $SO(10)$ symmetry gives up to relations between the down sector and the charged lepton masses. The underlining discrete symmetry gives a contribution (from the charged lepton sector) to the PMNS mixing matrix which is bimaximal. This gives a strong correlation between the down quark and charged lepton masses, and the lepton mixing angles. We obtain that the small entries $V_{ub}$, $V_{cb}$, $V_{td}$, and $V_{ts}$ in the CKM matrix are related to the small value of the ratio $\delta m^2_{sol}/\delta m^2_{atm}$: they come from both the $S_3 \times \mathcal{P}$ structure of our model and the small ratio of the other quark masses with respect to $m_t$.

## 1    Introduction

It is well know that there could be some theoretical relations between quark and lepton masses, however apparently Nature indicates that the lepton mixing angle should be completely uncorrelated to the quark mixing angles. Recent neutrinos experimental data show that in first approximation, the lepton mixing PMNS matrix is tri-bimaximal, i.e. the atmospheric mixing angle is maximal, $\theta_{13} \approx 0$ and the solar angle is $\theta_{12} \approx \arcsin(1/\sqrt{3})$. The tri-bimaximal matrix follow in natural fashion in models invariant under discrete symmetry like $S_3$ which is the permutation symmetry of tree object [1]. These motivations suggest us to consider discrete symmetries in extensions of the unified version of the SM. In literature are investigated both unified models based on extension of the standard model such us $SO(10)$ [2] symmetry with [3] or without [4] continuous flavor symmetries, and not unified models based on discrete symmetries [5]. Although some

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of them appear to be promising in understanding the flavor physics and unification [6] we are still far from an unitarity vision of the flavor problem [7]. Because the $S_3$ flavor permutation symmetry is hardly broken in the phenomenology, in this paper we study a model invariant under the $SO(10) \times S_2 \times \mathcal{P}$ group, where the $S_2 \times \mathcal{P}$ group is the discrete flavor symmetry. We analyse the phenomenological implication of such discrete symmetry on flavor physics and our aim is to construct a minimal renormalizable model which reproduce all the masses and mixing angles of both quarks and leptons. The $S_2 \times \mathcal{P}$ symmetry implies that the resulting mass matrices of the fermion are not general, but depending each one on 5 free parameters only. Together with the assumption that the two Higgs in **10** couple to fermions with a Yukawa matrix of rank one [5], we obtain that the left mixing matrices are all bimaximal with the remaining mixing angle small. This implies that the CKM is almost diagonal in the $S_2$ exact case. In our model the tri-bimaximal PMNS mixing matrix is achieved by rotating the low energy neutrino mass matrix. In a very surprising way we obtain that the small entries $V_{ub}$, $V_{cb}$, $V_{td}$, and $V_{ts}$ are related to the small value of the ratio $\delta m^2_{sol}/\delta m^2_{atm}$ (coming from both the $S_2 \times \mathcal{P}$ structure of our model and the small ratio of the quark masses with respect to $m_t$). On the other side when the $S_2$ symmetry is dynamically broken only the Cabibbo angle becomes relevant.

# 2 Our model

In $SO(10)$ all the fermion fields, with the inclusion of the right-handed neutrino, can be assigned to the **16** dimensional multiplet. We introduce the three possibilities to construct renormalizable invariant mass terms

$$\textbf{16 16 10}, \quad \textbf{16 16 120}, \quad \textbf{16 16} \overline{\textbf{126}} \tag{1}$$

where **10**, **120**, $\overline{\textbf{126}}$ are Higgs scalar fields. We consider the patter breaking of $SO(10)$ into the Standard Model through the Pati-Salam $G_{224}$ group. From the branching rules of $SO(10) \supset G_{224}$, it can be show that the non negligible Majorana mass term can arise only from the third interaction in (1) with the $\overline{\textbf{126}}$ scalar field.

We introduce a $\textbf{16}^i$ multiplet for each flavor $i$. We split the fermions into the $\{1, 2\}$, which are taken doublet under $S_2$, and $\{3\}$, $S_2$ singlet. We add two Higgs scalars $\overline{\textbf{126}}^\alpha$, and a **120**. We assume that the two fields $\overline{\textbf{126}}^\alpha$ form a doublet under $S_2$, and we write the $SO(10) \times S_2$ invariant Lagrangian

$$\begin{aligned} L^b_{\text{yuk}} &= \textbf{I}_{ij}\ \textbf{16}^i\ \textbf{16}^j\ \textbf{10} + \textbf{g}_{ija}\textbf{16}^i\ \textbf{16}^j\ \overline{\textbf{126}}^\alpha \\ &\quad + \textbf{A}_{ij}\textbf{16}^i\ \textbf{16}^j\ \textbf{120} + \text{h.c.} \end{aligned} \tag{2}$$

The flavor indices $\{i, j\}$ run over $\{1, 2, 3\}$, and the $\alpha$ over $\{1, 2\}$. We

introduce a parity operator $\mathcal{P}$ under which the fields transform as follow:

$$\mathcal{P}\mathbf{16}^\alpha = -\mathbf{16}^\alpha \qquad \mathcal{P}\mathbf{16}^3 = \mathbf{16}^3$$
$$\mathcal{P}\mathbf{126}^\alpha = \mathbf{126}^\alpha \qquad \mathcal{P}\mathbf{120} = -\mathbf{120}$$
$$\mathcal{P}\mathbf{10} = \mathbf{10}$$

The symmetric tensor $\mathbf{g}_{ija}$, and the antisymmetric matrix $\mathbf{A}$ are the most general $S_2 \times \mathcal{P}$ invariant and are given by

$$\mathbf{g}_{ij1} = \begin{pmatrix} b & d & 0 \\ d & e & 0 \\ 0 & 0 & f \end{pmatrix}, \mathbf{g}_{ij2} = \begin{pmatrix} e & d & 0 \\ d & b & 0 \\ 0 & 0 & f \end{pmatrix}, \quad \mathbf{A} = A \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

while, as it will be clarified in the next section, $\mathbf{I}$ will not be taken the most general symmetric matrix invariant under our flavor group. The coupling constants in $\mathbf{g}$, $\mathbf{A}$, and $\mathbf{I}$ are assumed to be small enough to avoid problem with respect the electroweak precision tests. They are all of the same order of magnitude.

The decomposition of the $\mathbf{10}$, $\mathbf{120}$, and $\overline{\mathbf{126}}$ representations under the group $SU_L(2) \times SU_R(2) \times SU_c(4)$ are

$$
\begin{aligned}
\mathbf{10} &= (2,2,1) + (1,1,6) \\
\mathbf{120} &= (2,2,1) + (1,1,\overline{10}) + (1,1,10) + (2,2,15) + \\
&\quad (1,3,\overline{6}) + (3,1,6) \\
\overline{\mathbf{126}} &= (\overline{3},1,\overline{10}) + (1,\overline{3},10) + (2,2,15) + (1,1,6)
\end{aligned}
$$

Under the same group the $\mathbf{16}$ decompose in $(2,1,4)_L$ and $(1,2,\overline{4})_R$. Then the Dirac mass terms decompose as follow

$$(2,1,4)_L \times (1,2,\overline{4})_R = (2,2,1) + (2,2,15) \tag{3}$$

and the Majorana mass terms are

$$(1,2,\overline{4})_R \times (1,2,\overline{4})_R = (1,3,\overline{10}) + (1,1,\overline{10}) \tag{4}$$

where in (4) we have neglected the terms containing the $\mathbf{6}$ of SU(4) which break the color symmetry. The Majorana mass cames from the $(1,\overline{3},10)$ component of $\overline{\mathbf{126}}$ and the Dirac mass cames from the (2,2,1) and (2,2,15) components respectively of the $\mathbf{10}$, $\mathbf{120}$ and $\overline{\mathbf{126}}$. We assume, by using the experimental constrains coming out from the electroweak precision tests of the Standard Model, that there are only two light Higgs doublets. In the mass bases for the Higgs, two of the *vev*s are assumed to be $\approx 100$ GeV ($k^u$ and $k^d$) and all the others *vev*s are much smaller the 100 GeV.

We are able now to write down the mass matrices of the quarks and

leptons that follow from the model given by the Yukawa interactions (2)

$$M^u \;=\; k^u\,\mathbf{I} + \mathbf{\Delta^u} + (q_s^u + q_{adj}^u)\mathbf{A} \tag{5a}$$

$$M^d \;=\; k^d\,\mathbf{I} + \mathbf{\Delta^d} + (q_s^d + q_{adj}^d)\mathbf{A} \tag{5b}$$

$$M^l \;=\; k^d\,\mathbf{I} - 3\,\mathbf{\Delta^d} + (q_s^d - 3q_{adj}^d)\mathbf{A} \tag{5c}$$

$$M^\nu \;=\; k^u\,\mathbf{I} - 3\,\mathbf{\Delta^u} + (q_s^u - 3q_{adj}^u)\mathbf{A} \tag{5d}$$

$$M^{\nu_R} \;=\; \mathbf{\Phi} \tag{5e}$$

where $k^{u,d}$ are the $vev$s of the two standard Higgs doublets of $(2,2,1)$ in **10**, the $q^{u,d}$ are the $vev$s in **120** and the index $s$ and $adj$ stand for $SU_c(4)$ singlet and adjoint representation [4].

The matrices $\mathbf{\Delta}^{u,d}$, and $\mathbf{\Phi}$ are

$$\mathbf{\Delta^{u,d}} = \begin{pmatrix} \mathrm{b}\delta_1 + \mathrm{e}\delta_2 & \mathrm{d}(\delta_1 + \delta_2) & 0 \\ \mathrm{d}(\delta_1 + \delta_2) & \mathrm{e}\delta_1 + \mathrm{b}\delta_2 & 0 \\ 0 & 0 & \mathrm{f}(\delta_1 + \delta_2) \end{pmatrix}^{u,d}$$

$$\mathbf{\Phi} = \begin{pmatrix} \mathrm{b}\phi_1 + \mathrm{e}\phi_2 & \mathrm{d}(\phi_1 + \phi_2) & 0 \\ \mathrm{d}(\phi_1 + \phi_2) & \mathrm{e}\phi_1 + \mathrm{b}\phi_2 & 0 \\ 0 & 0 & \mathrm{f}(\phi_1 + \phi_2) \end{pmatrix}$$

where $\delta_\alpha^{u,d}$ are the $vev$s of the $(2,2,15)$, and $\phi_\alpha$, are the $vev$s of $(1,\overline{3},10)$ component in the two $\overline{\mathbf{126}}^\alpha$s. In the case that $\delta_1^{u,d} = \delta_2^{u,d}$, and $\phi_1 = \phi_2$ than we obtain that the $S_2$ discrete symmetry is unbroken. However, as we will show in the following sections, this is not the choice taken by Nature. For example this case will give a wrong Cabibbo mixing angle. To obtain a good masses and mixing angles pattern we must require that $S_2$ is dynamically broken.

# 3   Our ansatz

Up to now, the only assumption we did is the fact that there is a factor 100 between the two kind of $vev$s. This allows us to fit the big top mass.

By studying our model we find that we have more freedom than what we need to reconstruct all the masses and mixing angles in quark and lepton sector. For this reason we assume that the $\mathbf{I}$ matrix is not the most general one under the $S_2 \times \mathcal{P}$ symmetry.

In fact, although the most general $S_2 \times \mathcal{P}$ invariant symmetric matrix is of the form

$$\begin{pmatrix} \mathrm{b} & \mathrm{d} & 0 \\ \mathrm{d} & \mathrm{b} & 0 \\ 0 & 0 & \mathrm{f} \end{pmatrix}.$$

We make the ansatz that the matrix $\mathbf{I}$ is given by

$$\mathbf{I} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The reason for this ansatz is related to the high value of the top mass. The **I** gives (under the assumption that the $k$'s are much bigger than all the other *vev*s) the top, bottom, and tau masses, and the hierarchy between these and the other masses is given by the $\Sigma/k$ and $q/k$ ratios. Maybe it is possible to justify our ansatz from a symmetry bigger than $S_2$ which constrains the matrix **I** (such as a modification of the $U(2)$ in [3]) but we will not investigate this point in this paper.

For simplicity, we rewrite the $\Delta$s (and equivalently the $\Phi$) matrices as

$$\begin{pmatrix} \Sigma_1 & \Sigma & 0 \\ \Sigma & \Sigma_2 & 0 \\ 0 & 0 & \Sigma_3 \end{pmatrix}$$

Notice that the $S_2$ symmetry implies $\delta_1 = \delta_2$ and then that $\Sigma_1 = \Sigma_2$. Moreover the entry $\{3,3\}$ is irrelevant (except that in $M^{\nu_R}$), because the presence of the $k$'s in the mass matrices in eqs. 5.

# 4   Charged leptons and down quarks masses

We know that at the unification scale the relation between the quark and lepton masses are [8]

$$m_\tau \approx m_b, \tag{6a}$$

$$m_\mu \approx 3m_s \tag{6b}$$

$$m_e \approx \frac{1}{3}m_d \tag{6c}$$

It is easy to see that, due to our structure of the mass matrices, we obtain automatically the relation (6b).

From the equations (5b) and (5c) we obtain the relation

$$3\,M^d + M^l \;=\; 4\,k^d\,\mathbf{I} + 4\,q_s^d\,\mathbf{A}\,. \tag{7}$$

If the **120** do not couple to the fermions eq. (7) gives wrong relation between lepton and quark masses. This is the reason of the introduction of the **120** Higgs fields in the Lagrangian (2). While we need the $SU(4)$ singlet of the **120** to obtain good relations between the charged lepton and down quark masses, in the follow we will assume that the *vev* of the $SU(4)$ adjoint into the **120** is negligible and we will omit it.

From the fact that all the other *vev*s are much smaller that $k^d$'s, and by assuming that for the moment $\Sigma_2^d = \Sigma_1^d$, we get that the eigenvalues of

$$\mathbf{M^d} = \begin{pmatrix} \Sigma^d + \Sigma_2^d & \Sigma^d & -q^d \\ \Sigma^d & \Sigma^d + \Sigma_2^d & -q^d \\ q^d & q^d & k^d + \Sigma_3^d \end{pmatrix}$$

are approximately

$$\{m_d, m_s, m_b\} = \left\{ \frac{\Delta^d}{k^d}, \quad \Sigma_2^d, \quad k^d \right\}.$$

where $\Delta^d$ is a function of the *vevs* given by

$$\Delta^d = 2\left((\Sigma_2^d)^2 - (q^d)^2\right) + \frac{1}{4}(\Sigma^d - \Sigma_3^d + \Sigma_2^d)^2 + (2\Sigma^d + \Sigma_2^d)k^d . \quad (8)$$

Equivalently the eigenvalues of charged leptons matrix

$$\mathbf{M^l} = \begin{pmatrix} -3\Sigma^d - 3\Sigma_2^d & -3\Sigma^d & -q^d \\ -3\Sigma^d & -3\Sigma^d - 3\Sigma_2^d & -q^d \\ q^d & q^d & k^d - 3\Sigma_3^d \end{pmatrix}$$

are approximately

$$\{m_e, m_\mu, m_\tau\} = \left\{ \frac{\Delta^l}{k^d}, \quad -3\Sigma_2^d, \quad k^d \right\},$$

where $\Delta^l$ is another function of the *vevs*. It is obvious that the experimental relations (6) can be easily reproduced in our model. This fix the value of the $\Sigma_2^d$ (the eigenvalue of $M^l$ which is three times the eigenvalue of $M^d$) to $m_\mu$ at the unification scale, and $k^d$ gives the value of $m_\tau$ (by neglecting $\Sigma_3$, the third eigenvalues of $M^d$ and $M^l$ are equal). Notice that, in spite the relations between $\Delta^l$, and $\Delta^d$ (but this point should be better investigate, in fact it could be an evidence for a more fundamental symmetry of the Standard Model) needed to reproduce the electron and down masses, up to now, we fitted six experimental masses by using four *vevs*.

# 5 Lepton mixing angles and structure of the neutrino mass matrices

In general the lepton mixing matrix is $V_{PMNS} = U_{lL}^\dagger U_{\nu L}$, where $U_{lL}$ and $U_{\nu L}$ enter into the diagonalization of the charged leptons and neutrino mass matrices. It is straightforward that if charged leptons mass matrix has the general $S_2$ invariant structure then the $U^l$ matrix has the form [1]

$$\begin{pmatrix} -\frac{1}{\sqrt{2}} & a & b \\ \frac{1}{\sqrt{2}} & a & b \\ 0 & N_a & N_b \end{pmatrix} \quad (9)$$

With a mass matrix

$$\begin{pmatrix} 67.86 & 57.2 & 65 \\ 57.2 & 47.06 & 65 \\ 83.2 & 83.2 & 1560 \end{pmatrix} \quad (10)$$

we obtain

$$\begin{pmatrix} 0.64 & -0.77 & -0.057 \\ -0.77 & -0.63 & -0.056 \\ 0.0072 & -0.079 & -0.997 \end{pmatrix} \quad (11)$$

This means that the charged electron mass matrix is diagonalized by

$$U_e \approx -U_{23}(\theta^e)Diag\{1,1,-1\}U_{13}(-\theta^e)U_{12}(2\theta^e + Pi/4)$$

where $\theta^e \approx 0.07$.

The neutrino mass matrix, which plays a role for the lepton mixing angles, is the one which comes out from the see saw mechanism, which in our model is of type I. In our model the neutrino mixing matrix is again of the form 9, but, being with an almost exact $S_3$ symmetry, with a column of all entries of order $\frac{1}{\sqrt{3}}$ given by the singlet under the $\{1,2,3\}$ permutation group. Moreover the remaining $S_2$ symmetry implies a column of type $\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}$.

With a mass matrix given by

$$\begin{pmatrix} 7.5 & 3.45 & 4.05 \\ 3.45 & 1.5 & 4.05 \\ 4.05 & 4.05 & 6.75 \end{pmatrix} \tag{12}$$

we obtain

$$\begin{pmatrix} -0.19 & -0.73 & -0.66 \\ -0.91 & -0.13 & -0.40 \\ -0.38 & -0.67 & -0.63 \end{pmatrix} \tag{13}$$

This means that the neutrino mass matrix is diagonalized by

$$U_e \approx -U_{23}(\pi/4 - \theta^\nu)Diag\{-1,1,1\}U_{13}(-\pi/4)U_{12}(\theta^\nu - Pi/2)$$

where $\theta^\nu \approx \arcsin(0.22)$.

We see that we obtain the tri-bimaximal PMNS mixing matrix

$$\begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{14}$$

which fit the experimental data [9].

# 6    Up quark masses

Let us now analyze the up quark mass matrix:

$$\begin{pmatrix} \Sigma^u + \Sigma_1^u & \Sigma^u & -q^u \\ \Sigma^u & \Sigma^u + \Sigma_2^u & -q^u \\ q^u & q^u & k^u + \Sigma_3^u \end{pmatrix}$$

His eigenvalues are approximately

$$\{m_d, m_c, m_t\} = \left\{ \frac{\Delta^u}{k^u}, \quad \Sigma_2^u, \quad k^u \right\}.$$

where $\Delta^u$ is a function of the *vev*s like (8). With $k^u$ we fit the experimental values of the top mass. By using the remaining freedom for the values of the *vev*s $\Sigma_2^u$ we fit the experimental values of the charm quark masses. For the up quark mass we have two cases: if $q$ is small compared to $k^u$, then there is a fine tuning between $\Sigma^u$ and $\Sigma_2^u$. If $q$ is bigger there is a fine tuning which fix the ratio $\Sigma^u/q^u$. In our fit will use the first situation, and impose that $q$ is much smaller then $k$.

# 7   Neutrino masses and the CKM matrix

The low energy neutrino masses, coming from the see-saw between the Dirac and Majorana neutrino mass matrices, depend directly on the $k^u$, the three $\Sigma_i^u$, and the four *vev*s $\phi$. The small value of the ratio $\delta m_{sol}^2/\delta m_{atm}^2$ is approximately equal to $-2(q^u)^2/(k^u)^2$. This fact is coming from both the $S_2$ structure of our model and the small ratio between the other quark masses and $m_t$.

As we told, if the $S_2 \times \mathcal{P}$ symmetry is exact than the CKM matrix is not the right one. The $S_2 \times \mathcal{P}$ symmetry in our model implies that the left mixing matrices are all bimaximal with the remaining mixing angle small. This implies that the CKM is almost diagonal in the $S_2$ exact case.

We observe that the small entries $V_{ub}$, $V_{cb}$, $V_{td}$, and $V_{ts}$ in the CKM matrix are related to the small value of the ratio $\delta m_{sol}^2/\delta m_{atm}^2$. All of them, in our model, are approximately proportional to a power of $q^u/k^u$.

In our model, the $S_2$ symmetry is broken only in the neutrino-up sector to fit the CKM mixing angles and to not destroy the prediction of a bimaximal PMNS mixing matrix. In this case, the Cabibbo angle is the only mixing angle hardly related to the $S_2$ breaking. Moreover this breaking introduce a correction for the other entries of the CKM which goes into the right direction for obtaining $V_{ub} << V_{cb}$, and $V_{td} << V_{ts}$. Finally we get the following solution for the CKM matrix

$$\begin{pmatrix} 0.9742 & 0.226 & 0.0036 \\ 0.225 & 0.9735 & 0.039 \\ 0.012 & 0.038 & 0.9992 \end{pmatrix}$$

which agrees very well with the experimental values [10].

The $S_2$ breaking enters now in the determination of the $\theta_{sol}$ too. However we are able to impose that the low-energy neutrino mass matrix is diagonalized by a rotation into the $\{1, 2\}$ family, by using the freedom in the right-handed sector. In this way it is possible to fit both the experimental constraints about the value of $\delta m_{atm}^2$ and $\delta m_{sol}^2$, and the observed PMNS mixing matrix given in eq. (14). However to explore the full predictivety of our model we need a Monte Carlo simulation.

# 8 Conclusions

In this paper we analysed a model based on $SO(10)$ gauge symmetry times and $S_3 \times \mathcal{P}$ discrete flavor symmetry. The aim of this work was to show that there is a symmetry beyond the lepton and quark masses despite the fact that the CKM and PMNS matrix are so different.

By using the most general $S_2 \times \mathcal{P}$ invariant Lagrangian with one **10**, one **120**, and two **126** Higgs, we are able to reproduce all the quark and lepton masses and mixing angles. Moreover, by making an ansatz which allows us to reduce the number of free Yukawa coupling we are able to construct a model which predict the usual unification relations between the down and the charged lepton masses. Our model agree very well with the recent neutrinos experimental data, that in first approximation give the lepton mixing PMNS matrix tri-bimaximal (i.e. the atmospheric mixing angle is maximal, $\theta_{13} \approx 0$, and the solar angle $\theta_{12} \approx \arcsin(1/\sqrt{3})$). This tri-bimaximal matrix follow in natural fashion in our model. The $S_2 \times \mathcal{P}$ symmetry, together with the assumption that the two Higgs in **10** couple to fermions with a Yukawa matrix of rank one, implies that the left mixing matrices are all bimaximal with the remaining mixing angle small. This implies that the CKM is almost diagonal in the $S_2$ exact case.

By giving as input the three charged lepton masses and the down quark mass, we obtain as output the right values for the strange and bottom masses. Moreover we predict that the atmospheric mixing angle is maximal, and $\theta_{13} \approx 0$ lepton mixing angle.

By using the value of the top, charm and up quark masses we predict a small value for $\delta m^2_{sol}/\delta m^2_{atm}$ and for the entries $V_{ub}$, $V_{cb}$, $V_{td}$, and $V_{ts}$. Due to a property coming from the $S_2$ structure of our model, they are all related to the small value of the ratio of the other quark masses with respect to $m_t$. On the other side when the $S_2$ symmetry is dynamically broken the Cabibbo angle become relevant.

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